

Can the dark matter halo be a collisionless ensemble of axion stars?

J. Barranco,^{1,*} A. Carrillo Monteverde,^{1,†} and D. Delepine^{1,‡}

¹*Division de Ciencias e Ingenierías, Universidad de Guanajuato,
Campus Leon, C.P. 37150, León, Guanajuato, México.*

(Dated: December 12, 2012)

If dark matter is mainly composed of axions, the density distribution can be non uniform distributed but clumpy instead. By solving the Einstein-Klein Gordon system of a scalar field with a potential energy density of an axion like particle, we obtain the maximum mass of the self gravitating system made of axions called axion stars. The collision of axion stars with neutron stars may release the energy of axions due to the conversion of axions into photons in the presence of the neutron star magnetic field. We estimate the energy release and shown that it may exceed the solar luminosity per collision but should be much less than previous estimates [1, 2]. Future data from femtolensing should strongly constrain this scenario.

PACS numbers: 95.35+d, 14.80.Va, 04.40.-b, 98.80.Cq, 98.70.Rz

I. INTRODUCTION

Nowadays one of the most interesting problems in physics is to reveal the nature of the dark matter (DM). The existence of DM is supported by large amount of astrophysical observations, in both galactic and cosmological scales [3, 4]. Since those effects are purely gravitational, the standard candidate is a massive particle with null or very weak interaction with the rest of the particles of the standard model of particles (SM). Two of the leading candidates for dark matter are the axions and weakly-interacting massive particles (WIMPs) as neutralinos [5].

The axion is the pseudo Nambu-Goldstone boson generated in the spontaneous breaking of the $U(1)$ Peccei-Quinn (PQ) global symmetry [6–8]. PQ symmetry was introduced to explain the smallness of the strong CP violation in QCD. Initially, this new field was assumed to be "invisible" as very weakly interacting but in 1983, it has been shown that axion couples to two photons through Primakoff effects [9] and could be detected through its conversion into a photon in presence of a strong magnetic field [10].

Most of the direct detection methods for the axion are based on the conversion of axions into photons [11–13]. But it is also possible to constrain the axion mass and coupling through astrophysics observation. As usually, stars have strong magnetic fields. Sun or any other astrophysical object as dwarf stars, supernovae, etc. could produced axions in their core during cooling processes and to transform them into photons through Primakoff effects. The search for photons produced in stars through axion conversion can be used to put an upper limit on axion mass m_a and coupling f_a where f_a defines the scale of $U(1)_{PQ}$ symmetry breaking: $m_a \leq 10^{-3}\text{eV}$ and

$f_a \geq 10^9\text{GeV}$ [14–23]. Cosmological observations impose constraints on m_a to be bigger than 10^{-6}eV and $f_a \leq 10^{12}\text{GeV}$ [24–27]. Therefore, cosmological and astrophysical observations put a range on m_a to be

$$10^{-6}\text{eV} < m_a < 10^{-3}\text{eV}. \quad (1)$$

So, an axion mass within this range could be a nice candidate for Cold Dark Matter [28, 29] as it could form Bose-Einstein condensate [30].

The rate of detection needs a prescription on the dark matter distribution in the galactic halo, and in particular, the local dark matter density. There are arguments in favor of a non uniform distribution of dark matter in the galaxy [31–34]. For instance, early fluctuations in the dark matter may go nonlinear long before photon decoupling, producing density-enhanced dark matter clumps. The evolution of the axion field at the QCD transition epoch may produce gravitationally bound miniclusters of axions [35, 36]. Such minicluster, due to collisional $2a \rightarrow 2a$ process, may relax to a selfgravitating system [37, 38]. Those self-gravitating systems made of axions are simply called axion stars (AS). Previously, some authors explored the possibility that such axion stars may have some observable consequences. For instance, the collisions between axions stars and neutron stars may induce gamma ray burst [39], ultra-high energy cosmic rays [40], or induce emission of X-ray by neutron stars [1]. All those observables have in common the possibility that the axion may oscillate into a photon.

In this paper, we study the self-gravitating system made of axions with a realistic potential for the scalar field as in [41] contrary to [1, 39, 40]. In section II, we obtain the typical Mass-density curve varying the central density. This curve exhibits a maximum mass which depends on the value of the axion mass and the central density. Section III is dedicated to update signatures of the AS in the Galactic Halo due to AS collisions with neutron stars (NS). The huge magnetic field in NS, $B \sim 10^{10}$ Gauss, may convert to photons all axions involved in the collisions. In this section, an estimate of the total radiated energy is computed. Finally, we present our results

*Electronic address: jbarranc@fisica.ugto.mx

†Electronic address: alcarrillo@fisica.ugto.mx

‡Electronic address: delepine@fisica.ugto.mx

and compared to previous estimates in Section IV.

II. AXION STARS

A. On the formation of axion stars

The formation of axion miniclusters were studied in [35, 37, 38]. They did a numerical study of the evolution of a spherically symmetric axion fluctuation in an expanding universe at the QCD epoch, where the mass of the axion effectively switches on. The axion potential considered was

$$V(\phi) = m_a^2 f_a^2 \left[1 - \cos\left(\frac{\phi}{f_a}\right) \right], \quad (2)$$

where ϕ is the axion field, f_a the energy scale of decoupling, and m_a the axion mass. The study was done for a wide range of initial conditions and a common feature of the final density distribution is that it developed a sharp peak in the center, i.e. due to the attractive self-interaction, some non-linear effects induce axion perturbations that lead to very dense axion miniclusters. If such density peaks are high enough, gravitational collapse and subsequent virialization can occur. This relaxation is due to $2a \rightarrow 2a$ scattering and the associated relaxation time is smaller than the age of the universe if the energy density of the minicluster is $\rho > 10^{10} \text{eV}^4$ [37]. An improved numerical resolution of the axion mini-clusters revealed that around $\sim 10\%$ of the axion density will be in miniclusters with densities that can virialize to coherent axion fields on a self-gravitational well, i.e. axions stars [42].

Other mechanism that may lead the formation of axion miniclusters and dark matter miniclusters was proposed in [36]. There, at the time where the phase transition from a quark-gluon plasma to a hadron gas take place, the spectrum of density perturbations develops peaks and dips produced by the growth of hadronic bubbles. After wards, kinematically decoupled cold dark matter falls into the gravitational potential wells provided from those peaks leading the formation of dark matter clumps with masses $< 10^{-10} M_\odot$.

B. Properties of the axion stars

The previous picture on the formation of axion stars did not study the final self-gravitating system made of coherent axion fields. In order to do so, we have solved the Einstein-Klein-Gordon equation in the semiclassical limit

$$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle, \quad (3)$$

$$\left(\square - \frac{dV(\Phi)}{d\Phi^2} \right) \Phi = 0, \quad (4)$$

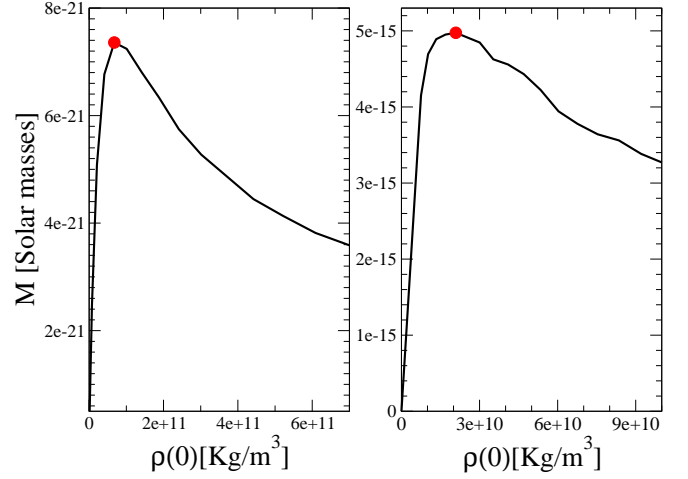


FIG. 1: Mass of axion star. Left Panel: $m_a = 10^{-3} \text{eV}$. Right Panel: $m_a = 10^{-5} \text{eV}$.

where the source of Einstein equations, the energy momentum tensor $\langle \hat{T}_{\mu\nu} \rangle$, is the average over the ground state of a real, quantized scalar field $\Phi(r, t)$ with potential given by the energy potential (2). We work in a spherically symmetric metric $ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(\sin^2 \theta d\phi^2 + d\theta^2)$, and to assume a harmonic dependence of the field $\Phi(r, t) = e^{i\omega t} \phi(r)$. The EKG system reduces to the following system of equations:

$$\begin{aligned} & a' + \frac{(1-a)a}{x} + (1-a)^2 x \\ & \times \left[\left(\frac{1}{\tilde{B}} + 1 \right) m_a^2 \sigma^2 - \frac{m_a \sigma^4}{12} + \frac{\alpha \sigma'^2}{1-a} + \frac{\sigma^6}{360} \right] = 0 \\ & \tilde{B} + \frac{a\tilde{B}}{x} - (1-a)\tilde{B}x \\ & \times \left[\left(\frac{1}{\tilde{B}} - 1 \right) m_a^2 \sigma^2 + \frac{m_a \sigma^4}{12} + \frac{\alpha \sigma'^2}{1-a} - \frac{\sigma^6}{360} \right] = 0 \\ & \sigma'' + \left(\frac{2}{x} + \frac{\tilde{B}'}{2\tilde{B}} - \frac{a'}{2(1-a)} \right) \sigma' + \\ & + \frac{1-a}{\alpha} \left[\left(\frac{1}{\tilde{B}} - 1 \right) m_a^2 \sigma^2 + \frac{m_a \sigma^3}{6} - \frac{\sigma^5}{120} \right] = 0 \end{aligned} \quad (5)$$

where we have defined $\phi = \frac{f_a}{\sqrt{m_a}} \sigma$, $\tilde{B} = \frac{m_a^2 B}{\omega^2}$, $r = \frac{m_p}{f_a} \sqrt{\frac{m_a}{4\pi}} x$, $a = 1 - A$ and $\alpha = \frac{4\pi}{m_a} \frac{f_a^2}{m_p^2}$, as it was done in [41]. In the present work we extend the results of [41] by solving the same system but for a wider range of masses of the axion field and many initial values of the axion field at the center $\sigma(x=0)$. Since the axion mass is restricted to be $10^{-6} \text{eV} < m_a < 10^{-3} \text{eV}$, we show in Fig. 1 the resulting masses of the AS as a function of the value of the central density $\rho(0) = m_a^2 \sigma(0)^2 / 2$. The mass-density plot shows that there exist a maximum mass for a particular value of the central density. The density profile of those maximum mass configurations are shown in Fig. 2 and the numerical values are reported in Table I. Those

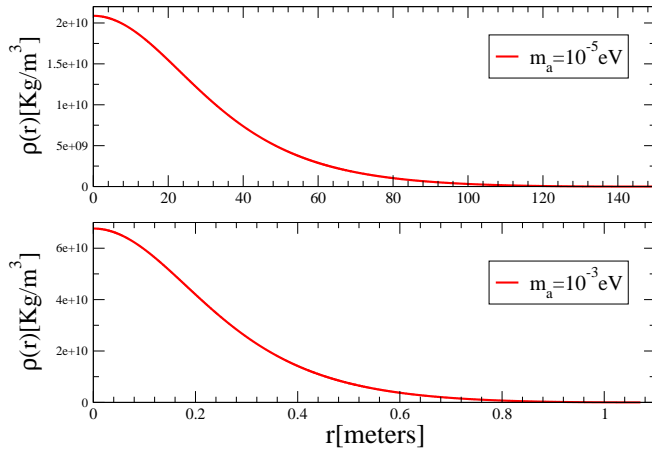


FIG. 2: Density profile. Top Panel: $m_a = 10^{-5}$ eV. Bottom Panel: $m_a = 10^{-3}$ eV.

TABLE I: Masses and typical radius R_{99} for the maximum mass configurations shown in Fig. 1 where R_{99} is the radius of the AS where 99% of the mass is contained

Axion mass (eV)	$\rho(0)$ (Kg/m ³)	Mass (M_\odot)	R_{99} (meters)
$m_a = 10^{-5}$	2.1×10^{10}	5.0×10^{-15}	119.40
$m_a = 10^{-3}$	6.8×10^{10}	7.4×10^{-21}	0.89

configurations are our benching mark in order to study possible astrophysical signatures.

III. POSSIBLE AXION STAR SIGNATURES IN THE GALACTIC HALO

The dark matter halo of galaxies can be considered as a collisionless ensemble of mini-MACHOs, as far as this hypothesis is not in contradiction with the limits imposed by microlensing or gravothermal instability. In particular, scalar field mini-MACHOS has been considered as a possible realization if their masses are below $10^{-7} M_\odot$ [43] to evade the microlensing limits. It turns out from our analysis on the maximum mass for an axion star, for $m_a = 10^{-5}$ eV that axions stars may play the role of such mini-MACHOS. In addition to microlensing, femtolensing could be a useful tool in order to look for axion stars [42]. Recently, new limits from femtolensing of gamma ray burst provides new evidence that primordial black holes in the mass range $10^{-16} - 10^{-13} M_\odot$ do not constitute a major fraction of the dark matter [44] and once again, the maximum mass of the axion star are consistent with those limits. In the case for an axion mass of $m_a = 10^{-3}$ eV, the maximum AS mass is far to be excluded from these current limits from femtolensing of GRBs since $M_{max} \sim 10^{-21} M_\odot$ but it is encouraging that in the future, femtolensing measurement may confirm or deny the possibility that the dark matter halo may be made of an ensemble of axion stars. In addition to AS, other types of dark matter clumps may play the role of

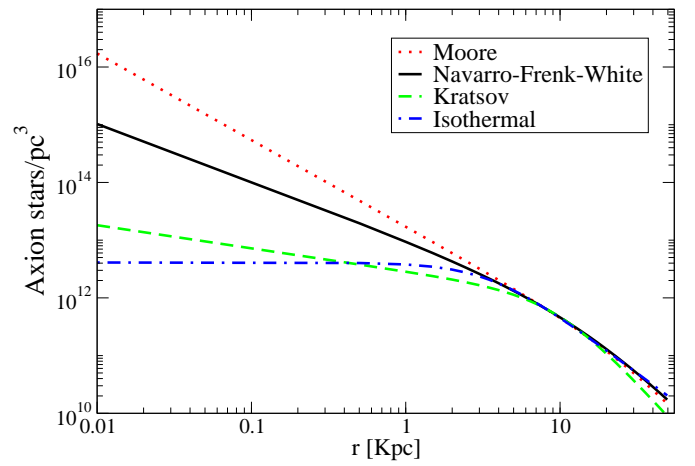


FIG. 3: Number of axion stars with a mass of $10^{-15} M_\odot$ for different dark matter profiles.

mini-MACHOS. In particular if dark matter is a non self-annihilating fermion, for some values of its mass it may fulfill the requirements to be a mini-MACHO [45]. For a self-annihilating fermion as the neutralino, neutralino stars [46–48], have been proposed. Actually, the maximum mass for a neutralino star is $\sim 10^{-7} M_\odot$ making them suitable dark matter mini-MACHOS candidates, although their stability is questionable [49].

The clumping of dark matter may change the expected number of events for direct dark matter searches since the local density may be largely affected at the vicinity of the Earth. Nevertheless, the clumpy dark matter may offer new ways for indirect detection. In the case of the axion, early studies of luminous axion clusters were done in the 80's [50]. In this work we reconsider the proposal of Iwazaki [39] where axion stars interact with the strong magnetic field of a neutron star producing photons, but now we consider the mass and radius of the AS obtained as general relativity indicates, by solving eqs. 5.

We can estimate the number of AS in a galactic halo using dark matter density profiles. In Fig. 3 it is shown the number of axion star with a mass of $10^{-15} M_\odot$ for four different DM density profiles: the Navarro Frenk White profile [51], the Moore [52], the Kravtsov [53], and the modified isothermal profile [54]. Notice the high number of axion stars at the center of the galaxy. Furthermore, it is expected that a galaxy has around $\sim .1\%$ of its total stars will finish their life as neutron stars. The Milky-Way, for instance, could have around $\sim 10^9$ neutron stars. The high number of axion stars (see Fig. 3) and the expected number of neutron stars implies a non-zero probability that AS collide with a neutron star and dissipate their energy under the effect of the neutron star strong magnetic field.

In order to estimate the amount of energy radiated by this process, we closely follow [39]. Starting with the

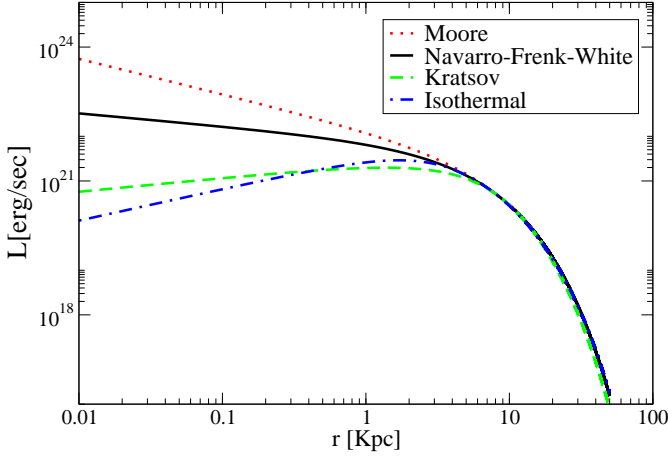


FIG. 4: Energy radiated by the collisions of axion stars with neutron stars in the dark matter halo of galaxies

Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial^\mu a \partial_\mu a - m^2 a^2) - \frac{1}{4} \frac{c\alpha}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad (6)$$

here c is a coupling constant of order unity, a is the axion field and $F^{\mu\nu}$ is the electromagnetic stress tensor and $\tilde{F}^{\mu\nu}$ its dual [10, 55]. And expressing this Lagrangian (6) in terms of the electric and magnetic fields \vec{E}, \vec{B} .

$$\mathcal{L}_{a\gamma\gamma} = \frac{c\alpha}{f_a} a \vec{E} \cdot \vec{B} \quad (7)$$

we can now derive a “modified” Gauss law, namely:

$$\partial \vec{E} = \frac{-c\alpha}{f_a} \vec{\partial} \cdot (a \vec{B}). \quad (8)$$

Thus, the axion field a may induce an electric field $\vec{E}_a = -c\alpha a \vec{B}/f_a$. If the neutron star has an electric conductivity σ , an electric current $J_m = \sigma E_a$ is produced. The dissipation in the magnetized conducting media, with average σ electric conductivity is evaluated using the Ohm’s law $W = \int_{AS} \sigma E_a^2 d^3x$, which, with help of the modified Gauss Law, can be directly related with the axion field and the neutron star magnetic field:

$$W = 4\pi \frac{c^2 \alpha^2 B^2 \sigma}{f_a^2} \int_0^R a(r)^2 r^2 dr. \quad (9)$$

Taking the two profiles for the axion field shown in Fig. 2, we perform the integration of eq. (9) numerically. The radiated energy is shown in Table II. Finally, we estimate the rate of collisions, such as we estimate the luminosity radiated every second. The number of collisions per pc^3 per second will be

$$R_c = n_{AS}(r) \times \rho_{NS}(r) \times S \times v, \quad (10)$$

where n_{AS} is the number of AS per pc^3 as a function of the distance to the galactic center (Fig. 1), ρ_{NS} is the

TABLE II: Energy radiated for an AS with the maximum mass configurations shown in Fig. 2 for a NS magnetic field $B = 10^8 \text{G}$ and an electric conductivity $\sigma = 10^{26} \text{s}^{-1}$. [57].

Axion mass (eV)	f_a (GeV)	Mass (M_\odot)	W (L_\odot)
$m_a = 10^{-5}$	6×10^9	5.0×10^{-15}	1.6×10^{10}
$m_a = 10^{-3}$	6×10^{11}	7.4×10^{-21}	2.4×10^4

probability to find a Neutron Star at that point, and we will assume this distribution to be [56]

$$\rho_{NS}(r) = A r^{\alpha-1} / \lambda^\alpha e^{-r/\lambda}, \quad (11)$$

where $\alpha \simeq 1.7$ and $\lambda(\text{kpc}) = 5.21$ as given in ref. [56]. For simplicity, the z -dependence is neglected. This assumption implies that NS distribution is assumed to be spherically symmetric. A is the normalization constant defined such that

$$10^9 \text{NS} = \int A \rho_{NS}(r) r^2 \sin^2 \theta dr d\theta d\phi. \quad (12)$$

Finally, S is the cross section which is simply computed as the effective area of the AS, i.e. $S = \pi R_{99}^2$. The velocity v of AS is assumed to be around $v = 2 \times 10^7 \text{cm/s}$.

The energy radiated per second is simply $L = W \times R_c$ which is shown in Fig. 4 for the case of the maximum mass of the axion star $M = 10^{-15} M_\odot$.

IV. CONCLUSIONS

As we can see from Table II, the energy dissipated by one collision of an AS with a NS is many orders of magnitude bigger than the solar luminosity. But our results for the total dissipated energy per second in NS-AS collisions are around 10 orders of magnitude less than previous estimates [1, 2]. As it can be seen from Fig 1, this difference comes from our computation of the maximal AS mass obtained through resolution of EKG equations. The obtained maximal AS mass is much smaller than the corresponding parameter used in ref. [1, 2]. It is important to notice that the maximal AS mass used in these references are now excluded by last observational results from Femtolensing of Gamma Ray Bursts [44].

Furthermore, improvement in femptolensing will help us to constrain the axion mass by constraining the maximum mass of AS. Observation of a sudden release of energy in Neutron stars may indicate the interaction of an AS with its strong magnetic field. If some of those signatures are observed, they can be considered as indications that the dark matter halo is made of an ensemble of axion stars. In our estimations, we have considered the largest mass of the AS and a hypothetical electric conductivity σ [57], nevertheless, we hope our results will encourage further studies in the possibility of detecting the axion by its possible clustering.

Acknowledgements

This work has been supported by CONACyT SNI-Mexico. The authors are also grateful to Conacyt

(México) (CB-156618), DAIP project (Guanajuato University) and PIFI (Secretaria de Educacion Publica, México) for financial support.

-
- [1] A. Iwazaki, Phys.Lett. **B486**, 147 (2000), hep-ph/9906353.
 - [2] A. Iwazaki, Phys.Rev. **D60**, 025001 (1999), hep-ph/9901396.
 - [3] G. Bertone, D. Hooper, and J. Silk, Phys.Rept. **405**, 279 (2005), hep-ph/0404175.
 - [4] E. Komatsu et al. (WMAP Collaboration), Astrophys.J.Suppl. **192**, 18 (2011), 1001.4538.
 - [5] e. Bertone, Gianfranco (2010).
 - [6] S. Weinberg, Phys.Rev.Lett. **40**, 223 (1978).
 - [7] F. Wilczek, Phys.Rev.Lett. **40**, 279 (1978).
 - [8] R. Peccei and H. R. Quinn, Phys.Rev.Lett. **38**, 1440 (1977).
 - [9] H. Primakoff, Phys.Rev. **81**, 899 (1951).
 - [10] P. Sikivie, Phys.Rev.Lett. **51**, 1415 (1983).
 - [11] P. Smith and J. Lewin, Phys.Rept. **187**, 203 (1990).
 - [12] I. Avignone, F.T., R. Brodzinski, S. Dimopoulos, G. Starkman, A. Drukier, et al., Phys.Rev. **D35**, 2752 (1987).
 - [13] K. Arisaka, P. Beltrame, C. Ghag, J. Kaidi, K. Lung, et al. (2012), 1209.3810.
 - [14] N. Iwamoto, Phys.Rev.Lett. **53**, 1198 (1984).
 - [15] G. Raffelt and D. Seckel, Phys.Rev.Lett. **60**, 1793 (1988).
 - [16] R. P. Brinkmann and M. S. Turner, Phys.Rev. **D38**, 2338 (1988).
 - [17] A. Burrows, M. S. Turner, and R. Brinkmann, Phys.Rev. **D39**, 1020 (1989).
 - [18] G. Raffelt and D. Seckel, Phys.Rev. **D52**, 1780 (1995), astro-ph/9312019.
 - [19] H.-T. Janka, W. Keil, G. Raffelt, and D. Seckel, Phys.Rev.Lett. **76**, 2621 (1996), astro-ph/9507023.
 - [20] H. Umeda, N. Iwamoto, S. Tsuruta, L. Qin, and K. Nomoto (1997), astro-ph/9806337.
 - [21] C. Hanhart, D. R. Phillips, and S. Reddy, Phys.Lett. **B499**, 9 (2001), astro-ph/0003445.
 - [22] G. G. Raffelt, J. Redondo, and N. V. Maira, Phys.Rev. **D84**, 103008 (2011), 1110.6397.
 - [23] E. Ferrer Ribas et al. (CAST Collaboration) (2012), 1209.6347.
 - [24] J. Preskill, M. B. Wise, and F. Wilczek, Phys.Lett. **B120**, 127 (1983).
 - [25] L. Abbott and P. Sikivie, Phys.Lett. **B120**, 133 (1983).
 - [26] M. S. Turner, Phys.Rept. **197**, 67 (1990).
 - [27] J. E. Kim and G. Carosi, Rev.Mod.Phys. **82**, 557 (2010), 0807.3125.
 - [28] P. Sikivie, Lect.Notes Phys. **741**, 19 (2008), astro-ph/0610440.
 - [29] J.-c. Hwang and H. Noh, Phys.Lett. **B680**, 1 (2009), 0902.4738.
 - [30] P. Sikivie and Q. Yang, Phys.Rev.Lett. **103**, 111301 (2009), 0901.1106.
 - [31] N. Dalal and C. Kochanek, Astrophys.J. **572**, 25 (2002), astro-ph/0111456.
 - [32] G. De Lucia, G. Kauffmann, V. Springel, S. D. White, B. Lanzoni, et al., Mon.Not.Roy.Astron.Soc. **348**, 333 (2004), astro-ph/0306205.
 - [33] J. Diemand, M. Kuhlen, P. Madau, M. Zemp, B. Moore, et al., Nature **454**, 735 (2008), 0805.1244.
 - [34] C. R. Keeton (2009), 0908.3001.
 - [35] C. Hogan and M. Rees, Phys.Lett. **B205**, 228 (1988).
 - [36] C. Schmid, D. J. Schwarz, and P. Widerin, Phys.Rev. **D59**, 043517 (1999), astro-ph/9807257.
 - [37] E. W. Kolb and I. I. Tkachev, Phys.Rev.Lett. **71**, 3051 (1993), hep-ph/9303313.
 - [38] E. W. Kolb and I. I. Tkachev, Phys.Rev. **D49**, 5040 (1994), astro-ph/9311037.
 - [39] A. Iwazaki, Phys.Lett. **B455**, 192 (1999), astro-ph/9903251.
 - [40] A. Iwazaki, Phys.Lett. **B489**, 353 (2000), hep-ph/0003037.
 - [41] J. Barranco and A. Bernal, Phys.Rev. **D83**, 043525 (2011), 1001.1769.
 - [42] E. W. Kolb and I. I. Tkachev, Astrophys.J. **460**, L25 (1996), astro-ph/9510043.
 - [43] X. Hernandez, T. Matos, R. A. Sussman, and Y. Verbin, Phys.Rev. **D70**, 043537 (2004), astro-ph/0407245.
 - [44] A. Barnacka, J. Glicenstein, and R. Moderski, Phys.Rev. **D86**, 043001 (2012), 1204.2056.
 - [45] G. Narain, J. Schaffner-Bielich, and I. N. Mishustin, Phys.Rev. **D74**, 063003 (2006), astro-ph/0605724.
 - [46] V. Berezhinsky, A. Bottino, and G. Mignola, Phys.Lett. **B391**, 355 (1997), astro-ph/9610060.
 - [47] L. Bergstrom, J. Edsjo, P. Gondolo, and P. Ullio, Phys.Rev. **D59**, 043506 (1999), astro-ph/9806072.
 - [48] J. Ren, X.-Q. Li, and H. Shen, Commun.Theor.Phys. **49**, 212 (2008), hep-ph/0604227.
 - [49] D.-C. Dai and D. Stojkovic, JHEP **0908**, 052 (2009), 0902.3662.
 - [50] T. W. Kephart and T. J. Weiler, Phys.Rev.Lett. **58**, 171 (1987).
 - [51] J. F. Navarro, C. S. Frenk, and S. D. White, Astrophys.J. **462**, 563 (1996), astro-ph/9508025.
 - [52] B. Moore, T. R. Quinn, F. Governato, J. Stadel, and G. Lake, Mon.Not.Roy.Astron.Soc. **310**, 1147 (1999), astro-ph/9903164.
 - [53] A. V. Kravtsov, A. A. Klypin, J. S. Bullock, and J. R. Primack, Astrophys.J. **502**, 48 (1998), astro-ph/9708176.
 - [54] L. Bergstrom, P. Ullio, and J. H. Buckley, Astropart.Phys. **9**, 137 (1998), astro-ph/9712318.
 - [55] G. Raffelt and L. Stodolsky, Phys.Rev. **D37**, 1237 (1988).
 - [56] A. Taani, L. Naso, Y. Wei, C. Zhang, and Y. Zhao, Astrophys.Space Sci. **341**, 601 (2012), 1205.4307.
 - [57] D. Baiko and D. Yakovlev, Astron.Lett. **21**, 709 (1995), astro-ph/9604164.